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Chemical Engineering Journal 103 (2004) 29–34

Chemical Engineering Journal

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Hydrodynamic scaling of a rectangular spouted vessel with a draft duct

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Received 8 January 2004; accepted 13 July 2004

Abstract

The scaling relationships developed by Glicksman et al. [2] for fluidized beds, have been modified to obtain scaling parameters for a rectangular spouted vessel operated with a draft duct. Similitude included matching the coefficients of restitution and friction of the particles. A CFD model was used to investigate the hydrodynamics of the “standard” model and hypothetical “small” (1/10th standard) and “large” (10× standard) sizes, while matching the dimensionless parameters for the spouted vessel. The results show good similarity at corresponding points for the three different sized vessels. In addition, a correlation is presented that can predict the solids circulation rate to within 17% of the CFD results for units with geometrical similarity to the system investigated.

Keywords: Reactor/contactor similarity scaling; Spouted bed with draft duct; Fluid-particle CFD model

1. Introduction

Scale-up or scale-down of fluid-particle contactors or reactors involving entrained or fluidized particles remains a challenging task. This is true for the authors’ specific application to electrolytic recovery of metals from aqueous solutions using spouted vessels of conductive particles, operated with a draft duct [1], where the variation of particle residence time in the moving bed cathode and the particle circulation rate with vessel size are of particular importance.

The primary objective of the current work is to modify and extend the scaling relationships proposed for fluidized beds by Glicksman et al. [2], to spouted vessels with draft ducts (tubes), and, in particular, to develop a correlation for the internal solids circulation rate. An evaluation of the modified “full” set of dimensionless parameters for units of varying size was performed by running CFD simulations rather than *via* the more classical approach with experimental tests in vessels of varying size, which would be prohibitively costly in our case. Even so, it is noted that the CFD model used for this purpose has been validated against experimental data [3,4]. This approach is similar to that used by Detamore et al. [5] for circulating fluidized beds.

The classical fundamental similarity or similitude scaling concept is that if two flow fields are geometrically similar and are operated with identical values of all the important independent dimensionless parameters, then the dependent dimensionless variables must also be identical at corresponding locations (e.g., see White [6]). Detamore et al. [5], using a CFD model, showed that for scaling of a circulating fluidized bed, not only the dimensionless parameters, but also the properties characterizing particle interactions (i.e., the coefficient of restitution and the coefficient of friction) must also be matched to ensure similarity. These findings are consistent with the current work.

2. Development for spouted vessels with draft ducts

Although circulating fluidized beds and spouted vessels share some common features, there are also some significant differences between them. Most notably, the annulus or peripheral region of a spouted bed is a moving packed bed with countercurrent interstitial flow of fluid, while all the particles in fluidized beds are fully supported by the fluid flow.

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doi:10.1016/j.cej.2004.07.004

Nomenclature

Ar	Archimedes number, Eq. (6).
d_p	particle diameter (m)
d_h	hydraulic diameter of the draft duct (m)
D	hydraulic diameter of the vessel (m)
e	entrainment distance (m) (Fig. 1)
e_{ss}	coefficient of restitution
Fr_p	Froude number based on the particle diameter, Eq. (5)
Fr	Froude number based on hydraulic bed diameter, Eq. (6)
g	gravitational acceleration (m/s^2)
G_s	solid circulation rate (kg/m^{2}s)
H	height of the spouted vessel (m) (Fig. 1)
H_d	height of the draft tube (m) (Fig. 1)
K_{sf}	exchange coefficient = K_{fs}, Eq. (3) (see Gidaspow, 1994)
L	depth of the spouted vessel (m) (Fig. 1)
L_d	distributor length (m) (Fig. 1)
P	pressure (Pa)
Re_p	particle Reynolds number, Eq. (5) (dimensionless)
U_0	inlet fluid velocity (m/s)
u_q	phase q velocity (m/s)
W	width of the spouted vessel (m) (Fig. 1)

Greek letters

α	angle of bottom inclination (Fig. 1)
α_q	volume fraction of phase q (dimensionless)
β	distributor angle (Fig. 1)
ϕ	angle of internal friction
γ	dimensionless entrainment distance (= e/D)
μ	viscosity (N s/m^2)
ρ_q	density of phase q (kg/m^3)
τ	shear stress (N/m^2)
δ	loading ratio (=V_{solid}/V_{total})

Subscripts

f	fluid
l	liquid phase
p	particle
s	solid phase

Since there is considerable particle–particle contact in the moving bed region of spouted beds, the rheological characteristics of the dense solid phase play a more dominant role than in circulating fluidized beds, where dense phase rheology is less important. Glicksman et al. [2] showed that the list of controlling dimensionless parameters could be reduced if fluid-particle drag is in either the viscous or inertial limit. However, given the very wide range of velocities encountered in spouted vessels, from very low in the moving bed, to very high in the spout or draft duct and entrainment region, “simplified” sets of scaling parameters are not expected to be very accurate for this application. Hence, the “full” set of scaling parameters was adopted and modified for spouted vessels.

He et al. [7] experimentally examined the parameters of Glicksman et al. [2] for a freely spouting bed and added two dimensionless parameters—the internal friction angle and the loose pack voidage to the original set of parameters. Costa and Tarano [8] applied the parameters of Glicksman et al. [2] to a 2D freely spouting bed, and derived correlations to predict values of maximum pressure drop, minimum spout velocity and maximum height of spouting.

In deriving the dimensionless groups, we take an approach similar to Glicksman et al. [2] by adopting the equations of motion for the fluid and particle phases in a spouted vessel of spherical particles. Therefore, for the fluid phase (as in Shirvanian et al. [3]):

$$\frac{\partial}{\partial t}(\alpha_f\rho_f\vec{u}_f) + \nabla_0(\alpha_f\rho_f\vec{u}_f \otimes \vec{u}_f) = -\alpha_f\nabla p + \nabla_0\bar{\bar{\tau}}_f + \alpha_f\rho_f\vec{g} + K_{sf}(\vec{u}_s - \vec{u}_f) \tag{1}$$

and for the solid phase:

$$\frac{\partial}{\partial t}(\alpha_s\rho_s\vec{u}_s) + \nabla_0(\alpha_s\rho_s\vec{u}_s \otimes \vec{u}_s) = -\alpha_s\nabla p - \nabla p_s + \nabla_0\bar{\bar{\tau}}_s + \alpha_s\rho_s\vec{g} + K_{fs}(\vec{u}_f - \vec{u}_s) \tag{2}$$

(See the Nomenclature list for definitions of the various symbols.) The dimensionless quantities are defined according to Glicksman et al. [2] as:

$$\vec{u}' = \frac{\vec{u}}{U_0};\ \nabla' = d_p\nabla;\ \ t' = \frac{U_0}{d_p}t;\ \bar{\bar{\tau}}' = \frac{\bar{\bar{\tau}}}{g\rho_s d_p} \tag{3}$$

From these expressions and the solids momentum balance, the controlling dimensionless parameters for the spouted vessel can be identified as:

$$\frac{K_{sf}d_p}{\rho_s U_0},\ \frac{gd_p}{U_0^2},\ \frac{\rho_f}{\rho_s},\ \frac{H}{d_p},\ \frac{D}{d_p},\ \frac{G_s}{\rho_s U_0} \tag{4}$$

plus bed geometry

Also, since the fluid–solid exchange coefficient is a function of the Reynolds number only (e.g., see Gidaspow [9]), the dimensionless parameter containing K_{sf} can be replaced by the Reynolds number.

The number of dimensionless independent parameters is fixed unless simplifications can be justified, but they can be arbitrarily combined and arranged in various forms. For instance, when $\rho_s \gg \rho_f$, the Archimedes number becomes:

$$Ar \approx \left(\frac{\rho_f U_0 d_p}{\mu}\right)^2\left(\frac{gd_p}{U_0^2}\right)\left(\frac{\rho_s}{\rho_f}\right) = \left(\frac{Re_{p_2}}{F_r}\right)\left(\frac{\rho_s}{\rho_f}\right) \tag{5}$$

As can be seen from the momentum equation for the solid phase, the solid pressure is calculated independently and used in the pressure gradient term ∇p_s. Because a Maxwellian velocity distribution is assumed in the CFD model [3] (see

Gidaspow [9]), a granular temperature is introduced that appears in the expression for solids pressure and viscosities [9]. The solids pressure is composed of a kinetic term and a second term due to particle collisions that is a function of the coefficient of restitution for particle collisions, e_{ss}, and particle–particle friction, as expressed by the coefficient of friction, $\tan(\phi)$, where ϕ is the angle of internal friction [10]. According to the kinetic theory treatment of liquid–solid systems, the complete set of dimensionless parameters should include those in Eq. (4) plus mechanical properties (also dimensionless) that characterize particle–particle and particle–wall interactions.

The dimensionless circulation rate, $G_s/\rho_s U_{0,}$ that appears in the list of independent dimensionless parameters proposed by Glicksman et al. [2] for fluidized beds, is difficult to match between different units for the spouted vessel system, since the internal circulation rate in such systems is not known a priori as a function of the other dimensionless parameters. Thus, here it is replaced by the solids loading ratio. Therefore, our *postulate* for the set of dimensionless parameters to be matched between spouted bed systems operated with draft ducts to provide similarity is:

$$Fr = \frac{U_0^2}{gD};\ Ar \approx \frac{\rho_s \rho_f d_p^3 g}{\mu^2};\ \frac{\rho_f}{\rho_s};\ \frac{d_p}{D};\ \delta = \frac{V_{\text{solid}}}{V_{\text{total}}} \tag{6a}$$

plus complete geometric similarity,

$$\frac{H}{W}, \frac{e}{H}, \frac{H_d}{H}, \frac{L}{W}, \frac{L_d}{W}, \alpha, \beta \tag{6b}$$

plus the same coefficients of restitution, e_{ss}, and friction, $\tan(\phi)$.

3. Validation of the scaling relationships

A rectangular spouted bed vessel was constructed (see Fig. 1) with plane walls, an inclined bottom, and an entry slot for liquid at the center of the vessel. The bed width, height, depth and entrainment lengths were 0.24, 0.4, 0.0254 and 0.035 m, respectively. The rectangular geometry was used to simplify experimental measurements and scale-up studies. The vessel interior was scanned with a five-hole, conical micro-pitot probe using an *x*–*z* translator, to measure the pressure and velocity of the liquid in the vessel at selected points [4]. A CFD model was also developed and the simulation results were validated against the experimental data [3].

While it is true that if our hypothesized list of scaling parameters were "exact," a numerical solution of the CFD model in dimensionless form would be applicable to spouted vessels of all sizes (as long as complete geometric similarity were maintained), there is no way of knowing a priori whether or not this is so, or to what degree this may be approximated. Therefore, the only method available to test the scaling hypothesis presented in Eq. (6), is to obtain "data" on spouted vessels of different size. Since an experimental approach would be prohibitively costly for the current application, numerical studies were performed *via simulation* to investigate the effect of the various dimensionless parameters on the performance of the spouted vessel, in a fashion similar to that in Detamore et al. [5]. A "standard case" was defined with the dimensions of the apparatus, an inlet liquid (water) jet velocity of 0.8 m/s, and 200 g of 1 mm diameter glass particles. The geometry and dimensions of the "standard case" are given in Fig. 1. The dimensionless parameters corresponding to the "standard case" were: $Fr = 1.42$, $Ar = 19800$, $\rho_f/\rho_s = 0.394$, $d_p/D = 0.022$ and $\delta = 0.045$. Furthermore, coefficients of restitution and friction were set to those obtained experimentally by Forester et al. [10] as $e_{ss} = 0.97$ and $\tan(\phi) = 0.092$, respectively. The model was then used to simulate geometrically similar spouted vessels with draft ducts with dimensional ratios of 1/10th (small) and 10× (large) times the "standard" geometry, with dimensionless parameters matched to that of the "standard case."

The resultant, calculated dimensionless circulation rates for the cases of small, standard, and large were 0.0195, 0.0194 and 0.023, respectively. These results show good similarity, within less than 1% difference for the small, and less than 20% for the large vessel, in comparison to the standard case. The larger error obtained for the large vessel is attributed primarily to computational error in that case. This effect is similar to that discussed by Detamore et al. [4]. The computational grid used ($22 \times 26 \times 4 = 2288$ for 1/4 of the physical domain, consisting of one-half the width and thickness of the rectangular spouted vessel) was suitable for resolving all the flow features for the "standard" case. This same grid was then used to model both the large (scaled-up) and small (scaled-down) systems. To maintain the same grid density for the large unit, would have required a grid of $220 \times 260 \times 40 = 2{,}288{,}800$ cells, which would have increased the computation time by about a factor of 1000. As it was, for 10 min of actual system operation of the large system, the simulation required approximately 10 days on a Pentium III machine. Therefore, since the objective was just to establish the utility of the proposed parameter set in providing similarity for spouted vessel units of varying size, it was decided to accept the error attributed to the "coarse" grid for the larger vessel, rather than performing a much more massive computation. The correlation developed below, however, does not suffer from this limitation since it is based on the "standard" case with the original grid density.

In Figs. 2–4 are presented samples of model predictions for profiles of the solid volume fraction and dimensionless solid and liquid velocities in the draft duct of the spouted vessel. Fig. 5 also shows the profile of the fluid volume fraction averaged over the cross section of the draft duct as a function of dimensionless height. As seen from these figures, all the variables examined exhibit good similarity between the "standard case" and the scaled models. This is interpreted as a strong indication of correct similarity analysis.

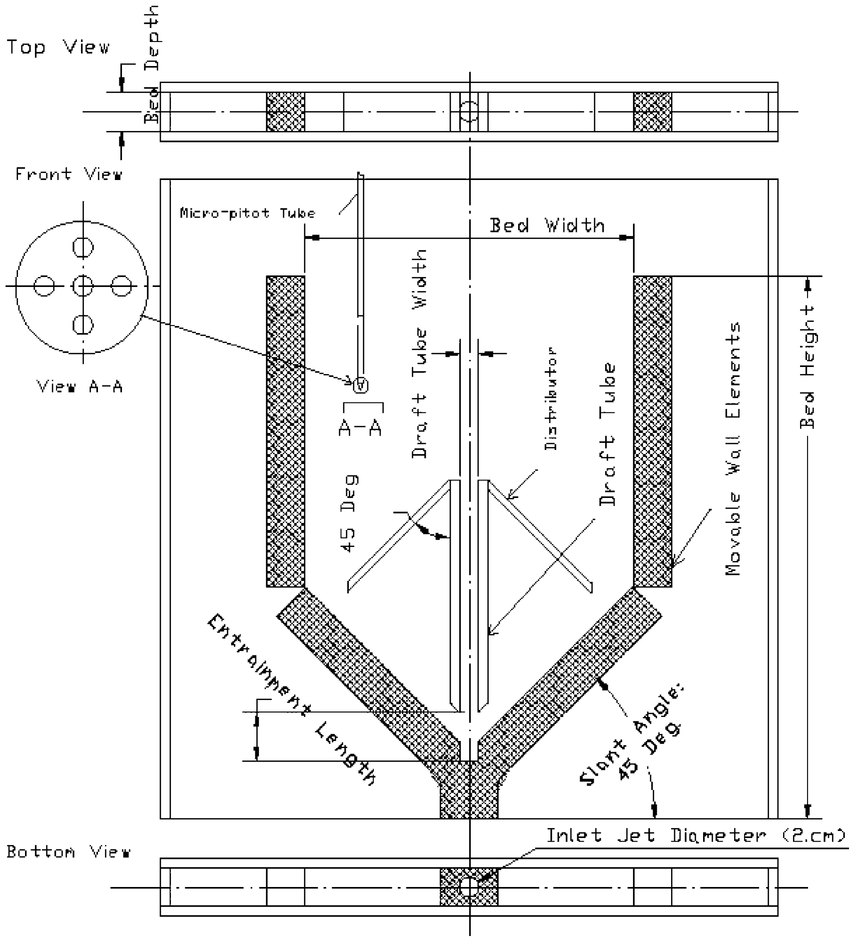

Fig. 1. Schematic of rectangular spouted vessel.

4. Development of the correlation

Based on the preceding establishment of similarity, a correlation was developed to predict the dimensionless solids circulation rate, $G_s/\rho_s U_0$, as a function of the independent dimensionless parameters listed in Eq. (6). For this purpose, each of the independent parameters was varied over a certain range, while keeping the others constant, and the solids circulation rate was determined. Ar, Fr, ρ_f/ρ_s, d_p/D and δ were varied over the ranges 10^3 to 10^5, 0.5–1.2, 0.33–0.67, 0.0054–0.043 and $(5.58 \times 10^{-3}) - 0.357$, respectively, selected to correspond to values of the dimensionless parame-

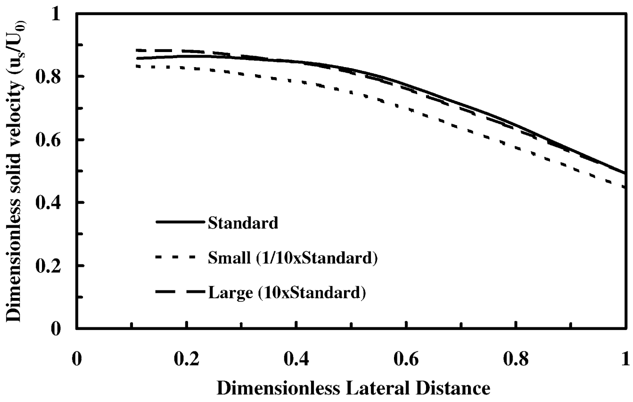

Fig. 2. Vertical component of particle velocity at the exit of the draft duct as a function of lateral distance from the center axis (0) to the wall (1) of the draft duct.

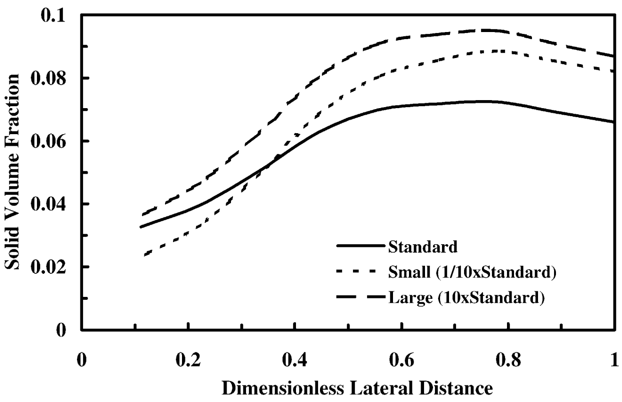

Fig. 3. Solid volume fraction profile at the middle of the draft duct as a function of lateral distance from the center axis (0) to the wall (1) of the draft duct.

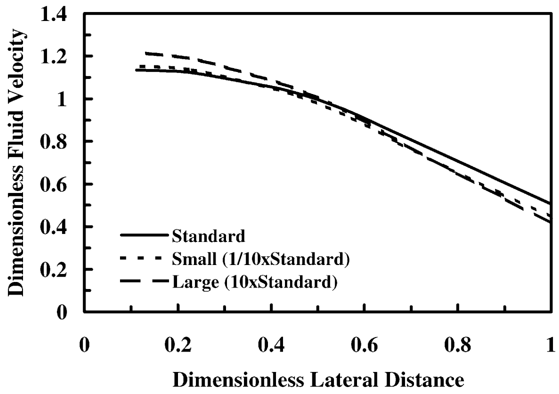

Fig. 4. Fluid velocity profile at the draft duct inlet as a function of lateral distance from the center axis (0) to the wall (1) of the draft duct.

ters that may be expected to be typically encountered in liquid spouted bed applications (such as that of the authors' application of electrolytic recovery of metals [1].) A nonlinear regression analysis yielded the correlation:

$$\frac{G_\mathrm{s}}{\rho_\mathrm{s}U_0} = 0.428Ar^{0.12}Fr^{-0.64}A_1A_2A_3 \tag{7}$$

where

$$\begin{aligned} A_1 &= 0.15 + 3.9\tfrac{d_p}{D} - 85.6\left(\tfrac{d_p}{D}\right)^2 \\ A_2 &= 0.19 - 0.22\exp\left(-0.0026\left(\tfrac{\rho_\mathrm{f}}{\rho_\mathrm{s}}\right)^{-10}\right) \\ A_3 &= 1.05 - \exp(-11.73\delta) \end{aligned} \tag{8}$$

This correlation is valid for liquid–solid spouted vessels with draft ducts with geometry similar to the system shown in Fig. 1; i.e., a spouted vessel with width:height:depth:draft duct width:entrainment length of 24:40:2.54:2:3.5, with a bottom inclination angle of 45°. The resultant correlation correctly predicts the solids circulation rate and the "critical loading" beyond which the solids circulation rate remains constant with increasing loading [3,4], with good accuracy.

As a test, this correlation was used to extrapolate the results for the two cases of $Ar = 10^7$ and $Fr = 5$, with all the other parameters corresponding to the "standard case." By comparing the results from the correlation and the simulation, it was found that the correlation predicted the circulation rate with deviations of less than 8 and 17%, respectively.

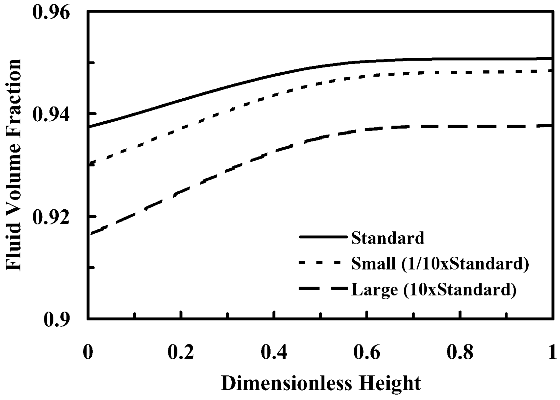

Fig. 5. Axial profile of radial mean fluid volume fraction in the draft duct as a function dimensionless height.

Although full geometrical similarity is a prerequisite for similarity analysis, with slight variations of the internal geometry of the spouted vessel, some useful correlations can be obtained that take into account the effects of entrainment length, e, and the hydraulic diameter of the draft duct, d_h. The resultant expression for 0.054 < e/D (=γ) < 1.09 and 0.39 < d_h/D < 0.63 is:

$$\frac{G_\mathrm{s}}{\rho_\mathrm{s}U_0} = 7.45A_1A_2A_3A_4Ar^{0.12}Fr^{-0.64}\left(\frac{d_\mathrm{h}}{D}\right)^{1.75} \tag{9}$$

where

$$\begin{aligned} A_1 &= 0.15 + 3.9\tfrac{d_p}{D} - 85.6\left(\tfrac{d_p}{D}\right)^2 \\ A_2 &= 0.19 - 0.22\exp\left(-0.0026\left(\tfrac{\rho_\mathrm{f}}{\rho_\mathrm{s}}\right)^{-10}\right) \\ A_3 &= 1.05 - \exp(-11.73\delta) \\ A_4 &= 0.7 - 0.6\gamma + 0.18\gamma^2 \end{aligned} \tag{10}$$

5. Conclusions

The scaling relationships, developed by Glicksman et al. [2] for fluidized beds were modified to obtain scaling parameters for a rectangular spouted vessel operated with a draft duct. Since particle–particle interactions significantly affect the hydrodynamics in a spouted vessel, it was also necessary to match the coefficients of restitution and friction for similitude. A CFD model that was previously developed and tested against experimental data by the authors [3] was used to investigate the hydrodynamics of the "standard" model and hypothetical "small" (1/10th standard) and "large" (10x standard) sizes, while matching the dimensionless parameters for the spouted vessel according to Eq. (6). The results show good similarity at corresponding points for the three different sized vessels, thereby validating the general robustness of the proposed similarity analysis. In addition, a correlation was developed that can predict the solids circulation rate to within 17% of the CFD results, with the limitation that it applies to units with geometrical similarity to the system investigated in the current work. The resultant correlation can be quite useful for sizing, designing, and estimating the solids circulation rate in scaled spouted bed systems without actually resorting to a CFD analysis.

Acknowledgements

This work has been supported by a grant from the US Environmental Protection Agency's Science to Achieve Results (STAR) program under Grant No. R82-6165.

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